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Supporting Information for

Material Transport across Europa's Ice Shell

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Introduction

The supporting information provided consists of one DOCX file for the Supporting Information and 3 supporting figures.

1. text01.docx is the supporting information text file consisting of 3024 words (including supporting figure captions). It is subdivided into four sections that detail the parameters used in the numerical models, the model setup, the proxy fluid approximation used as well as the description of the initial condition used in the model.
2. Figures S1, S2 and S3 are the three supporting figures that are used to demonstrate the proxy fluid approximation by noting the effects of varying the different parameters on our convection calculations.

Text S1

1. Parameters of the Model

Lists the parameters of the ice-ocean system setup for numerical models in this study.

2. **Model Setup**

   Describes the numerical model used in the convection experiments and the theory behind setting up these models.

3. **Proxy Fluid Approximation**

   Explains the concept and implementation of the proxy fluid as a substitute for the liquid water in the ocean layer and its implications on the study conducted and results obtained.

4. **Initial Condition**

   Description of the initial condition used in our convection study. This explains the technique used to obtain a stable convecting system, its advantages and challenges as well as implications for the results obtained.

1. Parameters of the Model

Several numerical models have previously studied convection in the ice-shell of Europa. The parameters of the system are based on measurements made during the planetary missions to Europa as well as on the data obtained from terrestrial glaciological studies and lab experiments on ice. Europa's H_2O layer is relatively thin (~100-170 km) compared to the radius (~1565 km) of the moon and hence high pressure ices are not expected to exist in its interior. Therefore, Europa's ice-shell is made of the hexagonal ice phase, ice Ih which is the ice phase prevailing on Earth. This has allowed us to extrapolate the properties of ice on Earth to higher pressures on Europa.

Our two phase model consists of pure ice and pure water. The parameters used in our study are consistent with previous convection studies of Europa. These parameters are listed in table 1 of the main manuscript.

Another parameter used in the model is the logarithm of the viscosity contrast across the ice shell which is represented by the activation coefficient denoted by *A*. We use the Newtonian viscosity formulation (Showman and Han, *J. Geophys. Res.*, 109, 2004) for water ice described by,

$$\eta = \eta_m \exp\left[A\left(\frac{T_m}{T} - 1 \right) \right]$$

Where η_m (Pa-s) is the melting viscosity of ice, *Tm* (K) is the melting temperature of ice which is defined as a function of pressure and *T* (K) is the temperature.

The activation coefficient *A* can be expressed as,

$$A = \ln\left(\frac{\eta_{\max}}{\eta_{\min}}\right)$$

Where η_{max} and η_{min} are the maximum and minimum values of viscosity of ice respectively.

Here *A* determines the temperature dependence of ice viscosity. In our cases, the value of *A* ranges from 5.0 to 13.8 corresponding to viscosity contrasts in the range ~10^2 - 10^6 across the ice-shell. The melting viscosity of ice, η_m is fixed in each calculation at 10^{16} Pa-s. However we have tested a range of melting viscosity values (~ 10^{18} – 10^{15} Pa-s) for cases not shown in this manuscript. The different melting viscosities affect the convective dynamics of ice shell thus changing the amount of new ice formed and transported. The ice convection would additionally be affected by viscosity of proxy fluid and the stability of the two-phase convecting system is adjusted by using numerical parameter ΔT. This is demonstrated in supporting figures 2 and 3 (for η_m = 10^{18} Pa-s) and further described in sections 3 and 4 of the supporting info.

In our model we adopt an ice viscosity that is dependent only on the temperature. The melting viscosity of ice for the cases presented in this study is slightly higher than the values generally adopted for Europa's ice-shell. For instance, the Showman and Han, 2004 study explores a range of values from 10^{12} Pa-s to 10^{14} Pa-s. Because our study employs a low-viscosity proxy fluid to represent the liquid water phase that is 1000x less viscous than the least-viscous ice, it becomes computationally prohibitive to reach these values while still maintaining adequate grid resolution. Therefore, we must use somewhat higher values for ice viscosity. For the same reason, the values of

A explored for our study are less than the value used by Showman and Han, 2004 study which is based on the Goldsby and Kohlstedt study (*J. Geophys. Res*., 106, 2001).

Model Setup

The main aim of our study is to perform simple numerical experiments in a two phase convecting system in order to investigate the possible dynamics of heat and mass transfer across the system and its implications for similar processes in Europa's putative ice-ocean system. Hence we setup a two phase model consisting of two convecting layers – a convecting ice-shell over a convecting ocean. The top of the system is the cold boundary fixed at the average surface temperature on Europa ~ 95 K. This top boundary refers to the ice-vacuum interface on Europa. The base of the system corresponds to the bottom of the ocean or the ocean-silicate boundary in Europa. The system is allowed to evolve self-consistently according to a phase law, however, in order to test our hypothesis two approximations are used: a.) proxy fluid approximation and b.) a prescribed temperature contrast, ΔT, across the system. These two numerical approaches are further explained in sections 3 and 4 of this supporting material. The first approximation employs a low viscosity proxy fluid instead of pure water for the ocean. This is done in order to deal with the extreme viscosity contrast between solid ice and liquid water which is computationally expensive to model. The second technique is to adjust a numerical parameter, ΔT which is the temperature difference between bottom and top of the system such that a stable two phase convecting system is attained. This method enables us to avoid the unwanted effects of bulk freezing or melting at the phase boundary while studying the formation and transport of new ice by convective plumes alone. Once a stable convecting system is achieved, we track the formation and transport of new ice from the phase interface into the warm ice plumes towards the surface by employing tracers. Technical details are expanded below.

Convection Model

We adapt the thermochemical version of the 2D mantle convection code, Citcom to the ice-water system in order to perform our investigation. Citcom solves the incompressible, dimensionless equations of conservation of mass, momentum and energy given below:

$$\nabla \cdot \vec{u} = 0$$

$$-\nabla P + \nabla \cdot (\eta \dot{\varepsilon}) = Ra(T - BC)$$

$$\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla) T = \nabla^2 T$$

where u is velocity, P is the dynamic pressure, η is the dynamic viscosity, $\dot{\varepsilon}$ is the strain rate, T is the temperature, C is the composition and t is the time. Ra and B are two non-dimensional parameters that control the vigor of convection and are defined as follows.

The thermal Rayleigh number, $Ra = \dfrac{\alpha \rho g \Delta T h^3}{\kappa \eta_m}$

where α, ρ, g, κ are system parameters as listed in table 1 of the main manuscript while η_m is the melting viscosity of ice, h is the thickness of the system and ΔT is the temperature difference between base and top of the system. The system refers to the entire H_2O layer. The base of the system refers to the base of the ocean (the ocean-silicate interface) and the top of the system refers to the ice surface atop the ice-shell (the ice-vacuum interface). We employed a thickness of the H_2O layer of 100 km for this study.

The density of the system is represented by the buoyancy ratio given by *B* as,

$$B = \frac{\Delta\rho}{\rho\alpha\Delta T}$$

where $\Delta\rho$ is the density contrast between liquid water and solid ice, ρ and α are material properties of ice as listed in table 1 of the main manuscript and ΔT is the temperature difference across the system (between the top of the ice shell and ocean-silicate surface).

The composition term *C* refers to the densities of the two phases in the system. In the ice-ocean system in our study, pure ice and pure water have same chemical composition. However, *C* identifies the density difference between the two phases of ice and water. This allows us to represent the two phases of ice and water in the phase panel of our figures.

Two Phase System

We incorporate the phase diagram of pure ice Ih from table 2 of Dunaeva et al. (*Solar Sys. Res.*, 44, 2010) into the model in order to achieve the two phase system. The equation for the phase law is given by,

$$T(P) = a + bP + c\ln P + d/P + e\sqrt{P}$$

a = 273.0159, b = -0.0132, c = -0.1577, d = 0.0, e = 0.1516 where T is the temperature in K, P is the pressure in bar and coefficients a, b, c, d and e are obtained from experimental data.

The phase – ice or water is dynamically assigned to the tracer according to the pressure dependence of temperature described above.

New Ice Formation

In an evolving two phase system, the thicknesses of the ice and fluid layer are continually varying until equilibrium is achieved. This involves bulk freezing or melting that is visible as a change in thickness of each layer. At a given temperature, convection initiates once a minimum thickness is attained. In an equilibrium case, ice formation is balanced by melt formation at the phase change interface between ice-shell and the ocean. That is, at the shell-ocean interface, it can be assumed that amount of ice formed by freezing is equal to the amount of water formed by melting which in turn is the definition of an equilibrated system. Since the study intends to investigate the formation and transport of new ice into the plumes at the interface, we initiate tracking of new ice only after the system is stably convecting and the ice-shell thickness is at equilibrium. Once the system is in equilibrium, we change the color of the new ice in the shell as it forms by freezing. This enables us to track the trajectory and density of new ice as it moves within the convecting shell. Tracers are used to represent the phase field i.e. the solid ice and the proxy fluid. We compute the fractional density of new ice formed for each element of the grid and map it across the phase panel of our calculations. The fractional density is defined as the ratio of new ice tracers to the total number of tracers in that element.

$$Fractional\ density = \left. \frac{n(tracers)_{new\,ice}}{n(tracers)_{total}} \right|_{element}$$

As the new ice formed is captured by the rising plumes and transported upwards, the color of the elements containing the new ice is changed according to the fraction of new ice present in it and this allows us to track the path of new ice in the convecting ice-shell.

2. Proxy Fluid Approximation

Liquid water has very low viscosity compared to solid ice. Since our calculations do not aim at modeling the dynamics of the ocean part of the system, we propose to introduce a proxy fluid in lieu of liquid water while still sufficiently approximating the ice system. The large viscosity contrast between solid ice and liquid water decouples the dynamics of the two layers in Europa, assuming that any possible phase gap between solidus and liquidus plays a negligible role in viscous coupling. We tune the viscosity of the proxy fluid in order to achieve this decoupling of convective dynamics. The proxy fluid thus has a viscosity higher than that of liquid water but still much lower than that of solid ice. We achieve this by continually decreasing the proxy fluid viscosity by an order of magnitude until we find that further decrease does not significantly affect the overall dynamics of the ice system (i.e. sufficient decoupling is achieved). We performed this exercise for different melting viscosities of ice (η_m ~10^{18} – 10^{15} Pa-s) and concluded that using a fluid viscosity value which is a thousandth of the melting viscosity of ice conveniently approximates the decoupling of the two layers in the real system. This means that any further decrease of proxy fluid viscosity does not affect the dynamics within the ice-shell considerably, and hence, we use the minimum requirement to increase the computational efficiency and make the problem tractable.

Viscosity contrast = Ratio of viscosity of proxy fluid to melting viscosity of the ice

$$\Delta\eta = \frac{\eta_{proxy\ fluid}}{\eta_m}$$

Supporting figure 1 illustrates how shell thickness for fixed values of η_m and ΔT depends upon the value of $\Delta\eta$. Shell thickness decreases with decreasing fluid viscosity for a given ΔT because the low viscosity fluid convects more vigorously (reducing the temperature difference across it) and the ΔT needs to be decreased to allow a thicker ice-shell to develop by reducing the heat into the system. Supporting figure 2 depicts the proxy fluid approximation in determining the upper limit of proxy fluid viscosity for which decoupling of convection in the two layers occurs. It can be noted that each time the fluid viscosity is decreased by an order of magnitude of ice viscosity and at a value equivalent to thousandth of the ice viscosity, sufficient decoupling is attained. Further decrease in viscosity (1/10,000) shown by the last model does not considerably change the dynamics of the convecting system. We have investigated this approximation for other values of melting viscosity of ice and concluded that a thousandth of the ice viscosity works as a suitable upper limit for approximating the decoupled convection in ice-ocean system analogous to Europa.

3. Initial Condition

All the calculations begin with an initial condition of a stable, convecting two phase system. The initial condition is achieved by allowing a fluid layer of thickness h to self-consistently evolve into a two phase convecting system as it cools from the top and is heated from the bottom. In order to achieve a convecting ice-shell over a convecting ocean layer and to maintain the stability of such a system, we adjust the value of the numerical parameter ΔT. ΔT is the temperature difference between the base and the top of the system. However, in the context of this study ΔT is merely a numerical parameter that is varied in order to attain a stable, two phase convecting system and does not have any physical significance. It is used to attain a stable convecting two phase system that can be used as an initial condition to study the formation and transfer of new ice across the convecting shell towards the surface.

For a fixed value of melting viscosity of ice (η_m) and viscosity contrast ($\Delta\eta$) across the ice shell, the value of ΔT was varied until decoupled convection in the ice-shell and the fluid layer is achieved. Convection in the ice-shell initiates only when a minimum threshold thickness is attained. Since shell thickness is sensitive to ΔT, we aim at attaining equally thick shell and fluid layers for simplicity. This is achieved by carefully varying the value of ΔT. Supporting figure 3 shows the effect of varying the ΔT for a system with fixed η_m and $\Delta\eta$. It can be noted here that increasing ΔT adds a higher amount of heat into the system and hence a thicker ocean layer is attained. Also, the vigor of the convection is higher for a higher ΔT. Supporting figure 2 shows the effect of changing the value of ΔT in conjunction with the varying viscosity of proxy fluid (refer section 3 of supporting material), in order to achieve decoupling of convection in the two layers. Here each of the five models shows that for different viscosity parameters, ΔT is varied such that a stable convecting system is achieved. ΔT is the temperature difference between the base and surface of the H_2O layer. For these experiments, we desire to maintain a steady-state ice-shell thickness, and we therefore tune the value of ΔT to achieve this. Higher and lower values of ΔT would result in bulk ice-shell melting and freezing with time, respectively. Each set of material parameters examined here (e.g., magnitude of viscosity, temperature dependence of viscosity, viscosity of the proxy fluid) requires a unique value of ΔT to achieve steady-state ice-shell thickness.

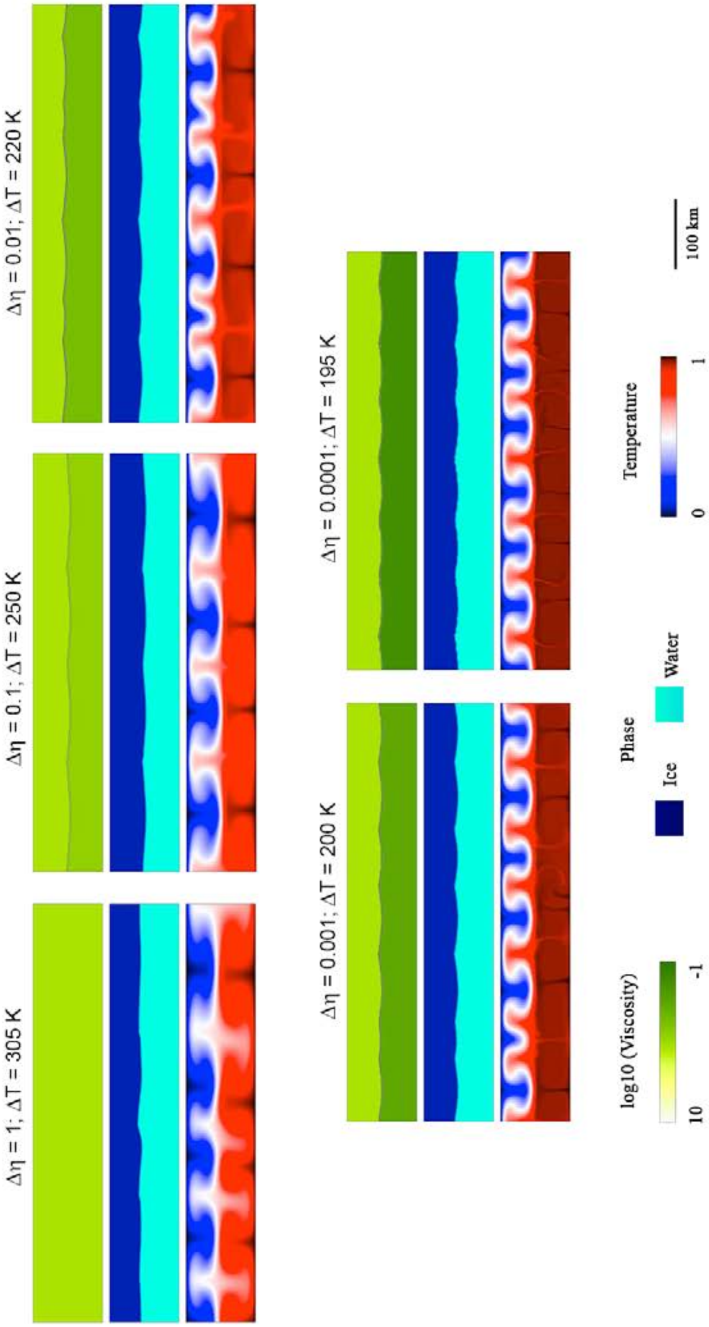
Δη = 1; ΔT = 305 K
Δη = 0.1; ΔT = 250 K
Δη = 0.01; ΔT = 220 K
Δη = 0.001; ΔT = 200 K
Δη = 0.0001; ΔT = 195 K
log10 (Viscosity)
10
-1
Phase
Ice
Water
Temperature
0
1
100 km

Figure S1 Effect of viscosity of proxy fluid on evolution of two-phase convection system. Four cases with different viscosities of proxy fluid are shown here. Each case has three panels – the top panel shows the logarithm of viscosity, the middle panel shows the phase with light blue representing proxy fluid and darker blue representing ice, and the bottom panel shows the temperature. The melting viscosity of ice (η_m) in all the four cases shown is fixed at 10^{18} Pa-s. The temperature contrast across the system represented by the numerical parameter ΔT is also fixed in all the cases. $\Delta\eta$ indicates the viscosity ratio of the proxy fluid to the solid ice. Hence, in the first case, the viscosities of the ice and proxy fluid are equal while in the last case, the proxy fluid is 1,000 times less viscous than the solid ice as indicated by a $\Delta\eta$ value of 0.001.

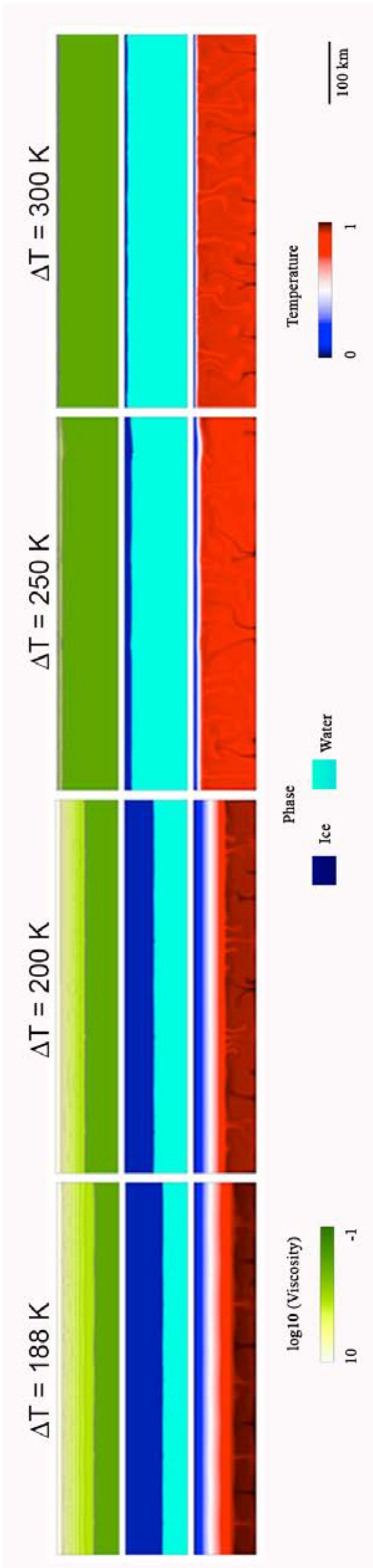
ΔT = 188 K
ΔT = 200 K
ΔT = 250 K
ΔT = 300 K
log10 (Viscosity)
10
-1
Phase
Ice
Water
Temperature
0
1
100 km

Figure S2 Exploring the low viscosity proxy fluid approximation. Five cases are shown here to demonstrate the low viscosity proxy fluid approximation in this study. Three panels are shown in each case. The top panel shows the logarithm of viscosity, the middle panel shows the phase – the light blue layer represents the proxy fluid while the darker blue phase represents the solid ice. The bottom panel represents the temperature. The melting viscosity of ice (η_m) in all the cases is fixed at 10^{18} Pa-s. The ratio of the viscosity of the proxy fluid to the melting viscosity of ice indicated by $\Delta\eta$ is varied such that from first case to the last case, the viscosity of proxy fluid is lowered by an order of 10 of the ice viscosity. Therefore, in the last case the viscosity of proxy fluid is 1/10,000th that of the ice viscosity. The numerical parameter ΔT, as explained in the text, is adjusted for each case such that a stable convecting system is attained.

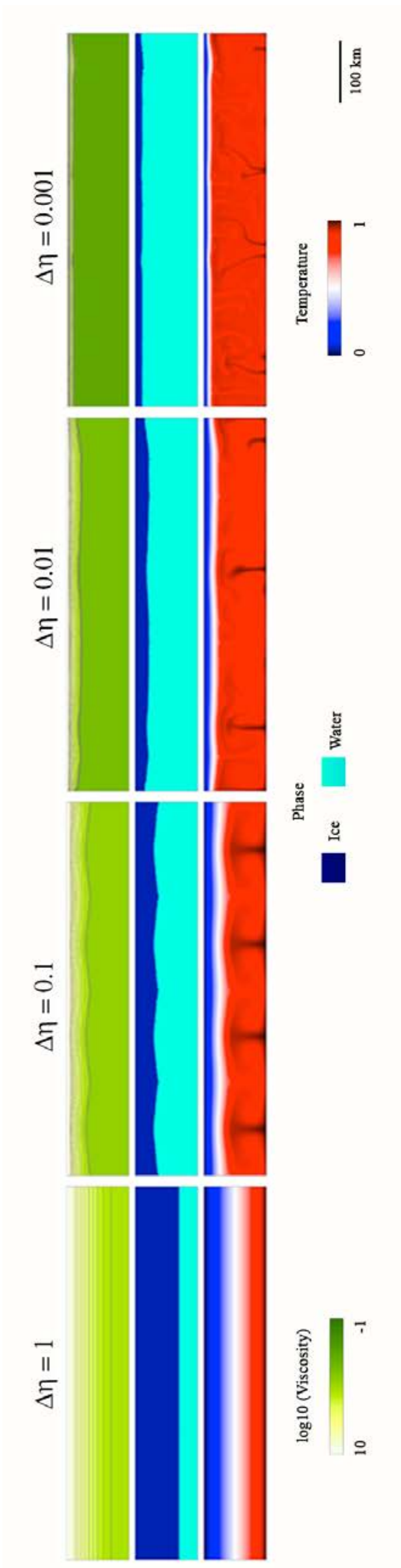
Δη = 1
Δη = 0.1
Δη = 0.01
Δη = 0.001
log10 (Viscosity)
10
-1
Phase
Ice
Water
Temperature
0
1
100 km

Figure S3 Demonstration of tuning the numerical parameter ΔT to achieve a stable convecting system. Four cases are shown here with a fixed melting viscosity of 10^{18} Pa-s and a fixed proxy fluid viscosity value equivalent to a 1/1,000th of the melting viscosity of ice. Each case has three panels – the top panel shows the logarithm of the viscosity, the middle panel shows the phase – the light blue represents the proxy fluid and the darker blue represents the solid ice. The bottom panel shows the temperature. The value of the temperature contrast across the system represented by the numerical parameter ΔT is changed in each case to demonstrate how this parameter is tuned in order to attain a stable convecting system. A stable convecting system can be defined as the two-phase convecting system where there is no bulk freezing or melting.